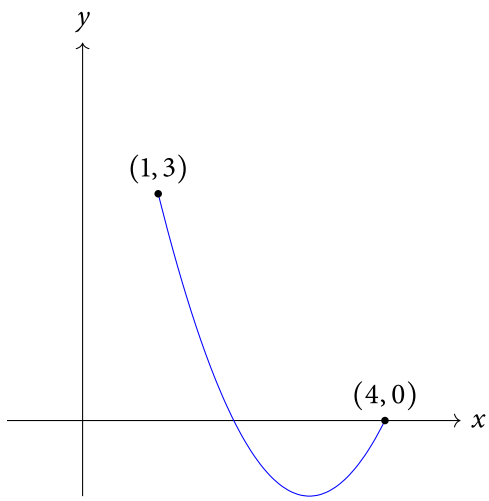
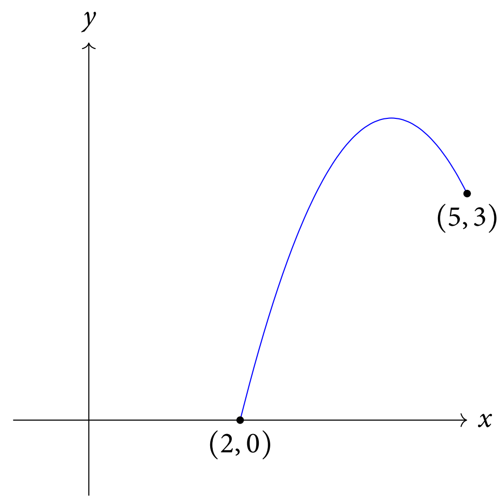
Math 1A: Calculus **Instructor: Alexander Paulin**

Quiz 2: §1.3, §1.4, and §1.5

Your name: *Discussions 201, 203 // 2018-09-07*

Problem 1 (3 + 2 = 5 points)**.** On the left, the graph of $y = f(x)$ is shown, for an unknown function f. The graph on the right has been obtained from the graph on the left through some sequence of transformations.





This problem has two parts.

(1) Give, in order, a list of *visual* transformations which transforms the left graph into the right graph. (In other words, give a list of instructions like "shift left by 4" etc.)

(2) Using the preceding part, express algebraically (in terms of f) the function whose graph is depicted on the right.

Solution:

(1) There are many ways to do this. Here's one method: shift right by 1, shift down by 3, and then reflect across the x-axis.

(2) Starting with $f(x)$ on the left, we shift right by 1 to get $f(x-1)$, then down by 3 to get $f(x-1)-3$, and finally reflect across the x-axis to get $-(f(x-1)-3) = \boxed{-f(x-1)+3}$.
The most common mistake in this part was writing $-f(x-1)-3$ instead. When you reflect across the x-axis, you are multiplying the *whole* function by −1. □

(Continued on back of sheet.)

Problem 2 (5 points)**.** Simplify the expression

$$\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2.$$

Solution: We expand and simplify.

$$\begin{aligned}\left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2 &= \frac{(e^x)^2 + 2e^x e^{-x} + (e^{-x})^2}{4} - \frac{(e^x)^2 - 2e^x e^{-x} + (e^{-x})^2}{4} \\ &= \frac{e^{2x} + 2e^0 + e^{-2x} - \left(e^{2x} - 2e^0 + e^{-2x}\right)}{4} \\ &= \frac{2}{4} - \frac{-2}{4} \\ &= \frac{1}{2} + \frac{1}{2} = \boxed{1}.\end{aligned}$$

□

Remark. This is the identity $\cosh^2(x) - \sinh^2(x) = 1$ of hyperbolic functions.

Problem 3 ($2 + 2 + 1 = 5$ points)**.**

(1) For $0 \le x \le 2$, let $f(x) = \log_2(3x/2 + 1)$. (I am specifying that the domain of f is the interval $[0, 2]$ for this problem.) What is the range of f?
(2) Find the inverse function f^{-1}. What are its domain and range?
(3) Compute $(f^{-1} \circ f \circ f^{-1})(1)$.

Solution:

(1) f is an increasing function on $[0, 2]$, so to find its range we just have to check the values of $f(0)$ and $f(2)$. These are $f(0) = 0$ and $f(2) = 2$, so the range is $\boxed{[0, 2]}$.
(2) If we write $y = \log_2(3x/2 + 1)$, then solving for x in terms of y gives

$$x = \frac{2}{3}(2^y - 1)$$

so the inverse function is given by

$$\boxed{f^{-1}(x) = \frac{2}{3}(2^x - 1).}$$

Since the range of the original function f was $[0, 2]$, $\boxed{\text{the domain of the inverse function } f^{-1} \text{ is } [0, 2]}$.
Also, $\boxed{\text{the range of } f^{-1} \text{ is } [0, 2]}$, because that was the domain of f. (In general, domain and range are interchanged for the inverse function.)
(3) $(f^{-1} \circ f \circ f^{-1})(1) = f^{-1}(1) = \boxed{2/3}$.

□